

Battery State Sensor

~Curve Fitting of Lead-acid Battery's Voltage Relaxation with Expanded Kalman-filter~

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ABSTRACT Stable open circuit voltage (OCV) prediction in a short time is required to detect the state of the lead-acid battery for automobile. We have found in our past development that the voltage relaxation behavior of the lead-acid battery could be expressed in higher-order exponential function of 5 or higher order, and the trial result to detect OCV in a short time was reported in Furukawa Denkou Jihou No.120 (Sept. 2007, in Japanese). In actual usage of the function, the optimum coefficients of the function shall be obtained according to the actual voltage behavior. This time, the calculation method is changed from the general least-squares method which requires high computational effort, to the expanded Kalman-filter calculation which is known as the less computational effort method, and the trial calculation was performed to assess the benefit.

1. INTRODUCTION

Because of the rapid popularization of start-stop system etc, the necessity for accurate automotive battery state sensing is increasing these days. In addition, the conventional technique of managing only the amount of power out of the battery by using a current sensor is no longer enough and a high functionality sensor which can detect the battery charge level and the degradation state in addition to the direct measurement values of such as current, voltage and temperature, are becoming a requirement. The development of the above described sensor was earlier promoted by German manufacturers such as Bosch, Hella and has started to be implemented in European vehicles since 2007 approximately. In case of Japanese car manufacturers, Honda started to equip with the Bosch made sensor in 2010. Furukawa Electric (FEC) has been in a position to be able to share knowledge with The Furukawa Battery Co., Ltd., and has been developing technology related to battery state since around 2000. From 2005, the development has focused on the similar in-vehicle sensor. As a result, the developed in-vehicle sensor was adapted, for the first time to Japanese car manufacturers in the HONDA ACCORD launched in North America on Sept. 19th 2012. Figure1 shows an example.



Figure 1 Furukawa Electric "Battery State Sensor".

The typical technology developed by FEC is the technology to detect OCV, which is an index to estimate the State of Charge (SOC), in a short time using a higher order exponential function. The bases of this technology was reported in Furukawa Denkou Jihou No.120¹⁾, (in Japanese). This technology can estimate OCV in a shorter time and with a higher accuracy in comparison to the conventional method, on the other hand a very high computational effort was required. Computational effort reduction was the issue for the in-vehicle sensor which has limitation in cost and dimension.

Until this point, prioritizing the certainty of the calculation, the development using mainly the least-squares method based on Levenberg-Marquardt method²⁾ which requires a high computational effort. The sensor for

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HONDA ACCORD, described before, based on this method. Meanwhile, (expanded) Kalman-filter method³⁾ is known to be used for the same purpose with less computational effort and, simple trial usage result has been reported in Furukawa Denkou Jihou No.120, (in Japanese). Here, full-fledged verification result of the capability for the Kalman-filter method is reported.

2. EQUILIBRIUM POTENTIAL OCV OF LEAD-ACID BATTERIES AND ITS RELAXATION BEHAVIOR

The formula (1) shows chemical changes associated with charging and discharging of the battery.



The left-hand side of the formula (1) indicates the discharging and the right-hand side indicates the charging. Here, the important issue is as follows. As a peculiar feature of lead-acid batteries, in its progress of discharging the sulfuric acid in the electrolyte is consumed and is replaced by water and the sulfuric density is changed. This shows that the electrolyte concentration can be a direct index to express the battery charging rate. Meanwhile, the equilibrium potential in oxidation-reduction reaction is expressed by Nernst equation⁴⁾ based on the thermodynamic equilibrium by using the activity of chemical elements for the reaction. Nernst equation for the lead-acid battery is shown in the formula (2).

$$E = E_0 + (RT/F) \cdot \ln(a_{H^+} \cdot a_{HSO_4^-} / a_{H_2O}) \quad (2)$$

E_0 : Standard electrode potential R : Gas constant
 T : Temperature F : Faraday constant
 H^+ : Hydrogen ion activity
 $a_{H_2SO_4^-}$: Sulfuric acid ion activity
 a_{H_2O} : Water activity

Activity means the proportion of each molecule occupied in the total number of molecules, and the mole fraction is used normally. E_0 is referred to as the standard electrode potential¹⁾ and corresponds to the case in which the activity of all the chemical elements is defined as 1 in the previously described Nernst equation. The value is calculated uniquely from Gibbs standard free energy of formation. In case of the lead-acid battery, the value is approximately 1.93 V⁴⁾ which can be considered a constant.

From the above facts, the battery charge level is expressed by the electrolyte concentration and the electromotive force of the batteries (in other words, the completely stable OCV without unhomogeneity in the electrolyte) can be expressed by mole fraction ($\hat{=}$ electrolyte concentration). In other words, the battery electromotive force can be an index to estimate the battery charge level.

This idea of associating the OCV with the battery

charge level of the lead-acid battery has been widely used as a very common method. But, there was a difficult problem to adapt to an application such as an automotive battery. Because, the charge and the discharge current always flows in driving the automobile, and even the battery voltage converges to stable OCV after the vehicle stops, the unhomogeneity of the electrolyte generated in a dynamic environment takes very long time such as ten and multiple tens of hours to eliminate this influence. In case of an automobile, the resting period of the automobile is different from drivers and from time to time condition. And we can not avoid saying that ensuring the stable and sufficient resting period is extremely difficult.

Figure 2 shows 86,400 s=24 hrs of the battery voltage behavior observation result. The battery voltage, which has received charging over-voltage, leads to a stable OCV, under the severe condition of -20°C. As this observation result shows, depending on the temperature, the battery voltage does not reach the stable OCV even after 24 hrs.

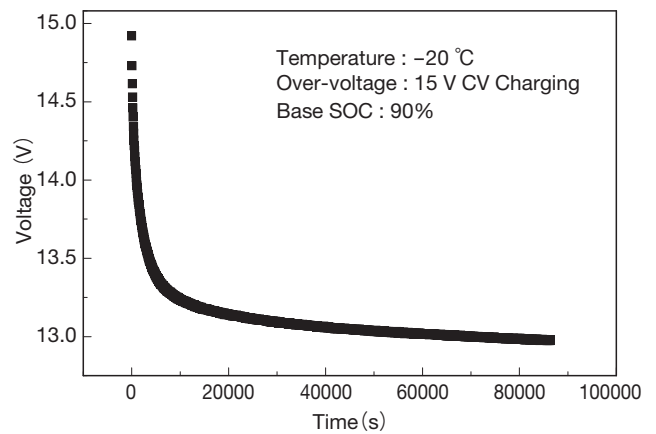


Figure 2 Relaxation of charging over-voltage.

3. FUNCTION TABLE OF RELAXATION BEHAVIOR OF OVER-VOLTAGE

We considered a method to predict the stable OCV from the short resting period that can be secured from a stable condition of the vehicle. The method expresses the relaxation behavior of over-voltage as a function of time and predicts the stable OCV as the convergence of voltage at an infinite time of this function. This study was conducted; and as a result, we have managed to obtain the expected results in 5-dimensional exponential function. This has been reported in Furukawa Denkou Jihou No.120, (in Japanese). The formula (3) represents the general formula of the 5th-order exponential decay function.

$$y = f(x) = y_0 + a_1 \cdot \exp(b_1 \cdot x) + a_2 \cdot \exp(b_2 \cdot x) + a_3 \cdot \exp(b_3 \cdot x) + a_4 \cdot \exp(b_4 \cdot x) + a_5 \cdot \exp(b_5 \cdot x) \quad (3)$$

The relaxation behavior of the over-voltage shown in Figure 2 was fit to a function by using the data analysis software on the market OriginPro7.5J. From Figure 3, it is clear that the higher the exponential dimension, the closer the function becomes to the measurement. The correlation coefficient R^2 of the 5-dimensional exponential function reaches 0.99998, and we can say that a good result obtained explains the behavior almost completely.

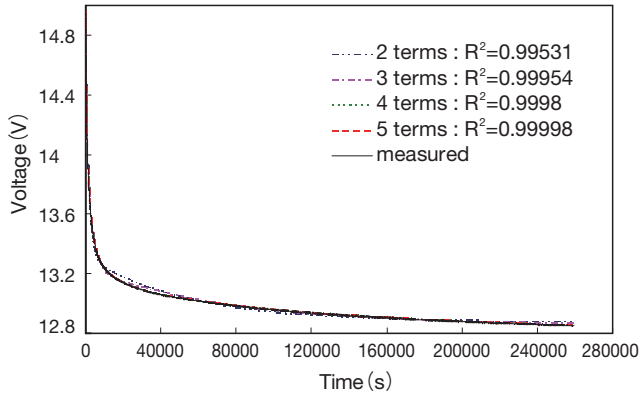


Figure 3 Results of high order exponential functions (72h).

4. THE LEAST-SQUARES METHOD AND THE KALMAN-FILTER CALCULATION

To adapt the above-mentioned knowledge for the prediction of the stable OCV, which is our objective, the relaxation behavior of the over-voltage shall be measured by in-vehicle a battery state detection sensor and the sensor is required to perform the similar curve fitting from the above mentioned OriginPro7.5. The least-squares calculation by the Levenberg-Marquardt method is used in the curve fitting engine of OriginPro7.5. The basic idea of the least-squares method is intended to obtain a minimum solution of a sum of squared deviation between measured data Y_n and function value $f(X_n)$ as shown in the formula (4).

$$\sum_{n=1}^N \{Y_n - f(X_n)\}^2 = \min \quad (4)$$

In case of the 5-dimensional exponential function, which is targeted in this study, solving the formula (4) means to solve the following 11 simultaneous equations (5) which correspond to 11 coefficients included in the formula (3).

$$\left. \begin{aligned} \sum_{n=1}^N \exp(b_1) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N \exp(b_2) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N \exp(b_3) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N \exp(b_4) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N \exp(b_5) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N X_n \cdot \exp(b_1) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N X_n \cdot \exp(b_2) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N X_n \cdot \exp(b_3) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N X_n \cdot \exp(b_4) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N X_n \cdot \exp(b_5) \cdot \{Y_n - f(X_n)\} &= 0 \\ \sum_{n=1}^N \{Y_n - f(X_n)\} &= 0 \end{aligned} \right\} \quad (5)$$

This type of nonlinear simultaneous equation does not have analysis solving method, except for a special exception. Iteration calculation, which gradually approaches the optimal solution from an initial value of an assumed solution, is a practical solving method. The Levenberg-Marquardt method is used most widely in these days for this iteration calculation. The general formula for updating the solution by the Levenberg-Marquardt method is shown in the formula (6).

$$\mathbf{u}^{(k)} = \mathbf{u}^{(k-1)} - (\mathbf{H}_u^{(k-1)} + cD[\mathbf{H}_u^{(k-1)}])^{-1} \times \nabla_u \mathbf{J}^{(k-1)} \quad (6)$$

Here, $\mathbf{u}^{(k)}$ is the vector of the coefficient to be obtained and the updating is repeated until the solution converges to the optimum value. $\mathbf{H}_u^{(k)}$ is the Hessian matrix, and a calculation formula is shown in the next formula (7). In this example, this is a matrix with 11x11 elements, 11x11matrix elements are calculated by partial differentiation in accordance with the 1 to N of individually obtained data and all of those are added.

$$\mathbf{H}_u^{(k)} = \sum_{n=1}^N \left\{ (\partial f(X_n) / \partial \mathbf{u}^{(k)}) \times (\partial f(X_n) / \partial \mathbf{u}^{(k)})^T \right\} \quad (7)$$

$D[\mathbf{H}_u^{(k)}]$ can be obtained easily by extracting only the diagonal elements, by completing the calculation of $\mathbf{H}_u^{(k)}$. But, an inverse matrix of $\mathbf{H}_u^{(k)} + cD[\mathbf{H}_u^{(k)}]$ which is the summation of both matrixes is obtained. The calculation of this inverse matrix is not easy. It is necessary to solve a simultaneous equation by using a high computational effort method such as the Gauss-Jordan method etc. $\nabla_u \mathbf{J}^{(k)}$ is a gradient and calculating formula is explained by the formula (8). This is also a vector with 11 elements including partial differentiation.

$$\nabla_{\mathbf{u}} \mathbf{J}^{(k)} = \sum_{n=1}^N \left\{ (Y_n - f(X_n)) \times (\partial f(X_n) / \partial \mathbf{u}^{(k)}) \right\} \quad (8)$$

Partial differential value of 11x11 elements for each observed data of the N shall be calculated and added together, and perform an inverse matrix calculation for the finally summed matrix of 11x11. Further more, the above calculation is repeated until a solution converges to the optimum value. As described, the Levenberg-Marquardt method is a very severe calculation to implement to the in-vehicle sensor which has a limitation in the manufacturing cost and the product dimension.

Meanwhile, the Kalman-filter calculation is well known as requiring much less computational effort method which can be adapted to the optimum coefficient estimation similar to the Levenberg-Marquardt method. The Kalman-filter calculation method was proposed by Hungarian-American Rudolf Kalman and used for the rocket orbital calculation of the Apollo program. This is the famous technology that contributed to the successful landing on the moon in Apollo 11. Currently, this method is widely used as the basic technology for the airplane automatic navigation and the car navigation systems. Initially, the filter theory proposed by Rudolf Kalman could only be adapted to the linear system (linear Kalman filter), and could not be adapted to non-linear system such as exponential function. In Furukawa Denkou Jihou No.120, (in Japanese), the linear Kalman-filter was simplified to be able to be used with assumption that power coefficients of exponential function can be calculated individually, and possible to be treated as constants in a function fitting, and the trial proceeded. The filter was improved to the expanded Kalman-filter which could be adapted to a non-linear system by Stanley Schmidt in NASA, in the stages of the Apollo program application. Nowadays, the filter is used more widely. Here, we tried to optimize all the coefficient of the high-order exponential function by using the expanded Kalman-filter. The general formula of the expanded Kalman-filter is shown in the following.

One step forward estimation:

$$\left. \begin{aligned} \hat{x}_n^- &= f(\hat{x}_{n-1}^+, \hat{u}_{n-1}^+) \\ \Sigma_{\hat{x},n}^- &= A_n \Sigma_{\hat{x},n-1}^+ A_n^T + \Sigma_n \end{aligned} \right\} \quad (9)$$

Jacoby matrix calculation:

$$\left. \begin{aligned} A_n &= \frac{\partial f}{\partial \hat{x}} \Big|_{\hat{x}_{n-1}^+} \\ C_n &= \frac{\partial h}{\partial \hat{x}} \Big|_{\hat{x}_n^-} \end{aligned} \right\} \quad (10)$$

Filtering calculation:

$$\left. \begin{aligned} \Sigma_{\hat{x},n}^- &= A_{n-1} \Sigma_{\hat{x},n-1}^+ A_{n-1}^T + \Sigma_w \\ L_n &= \Sigma_{\hat{x},n}^- C_n^T [C_n \Sigma_{\hat{x},n}^- C_n^T + \Sigma_v]^{-1} \\ \hat{x}_n^+ &= \hat{x}_n^- + L_n [y_n - h(\hat{x}_n^-, 0)] \\ \Sigma_{\hat{x},n}^+ &= [1 - L_n C_n] \Sigma_{\hat{x},n}^- \end{aligned} \right\} \quad (11)$$

\hat{x} : State vector

y : Observed value

A : Jacoby matrix in time evolution model

C_n : Jacoby matrix of observed model

$\Sigma_{\hat{x},n}$: Covariance matrix

L_n : Kalman gain

Σ_w : System noise

Σ_v : Observation noise

u_n : Input vector

When observed value y_n can be represented by the formula (12) with the state vector, and the state vector x_n is in space state representation as shown in the formula (13),

$$y_n = h(x_n) \quad (12)$$

$$x_n = f(x_{n-1}) \quad (13)$$

The purpose of this calculation is to proceed filtering (= optimization of state vectors) by using the formula (10) (11) for more optimum prediction with further observation by proceeding the one-step forward estimation using the formula (9).

Similar to the least-squares method, an initial value shall be set prior to the calculation starting; the largest difference is as follow. In case of the least-squares method, the N predetermined observation data shall be arranged and the optimization starts at the time of the buffering completion for all the data, but the Kalman-filter method proceeds the optimization at each observation procedure. In other words, the Kalman-filter method, in principle, does not need k repetition, and the buffers for observation data from 1 to N are not necessary. This method is the infinite response filter that is not subjected to restriction of N.

In comparison to the computational effort, the strict comparison is not possible since the computational effort is greatly different depending on the number of repetition k until convergence. With assumption that from 30 to 50 repetition is required, if optimization is completed by a set of learning from 1 to N that is the same observation number of the least-squares method, about two digits of computational effort reduction can be expected.

As a matter of fact, the least-squares method has excellent components. The least-squares method is superior in reliability and robustness as it can reach the optimum value more certainly. But, if the same accuracy of calculation is obtained by the Kalman-filter method, a significant reduction of computational effort can be expected as described above.

5. APPLICATION OF THE KALMAN-FILTER TO THE EXPONENTIAL CURVE FITTING

When applying the expanded Kalman-filter to the exponential curve fitting, the formula (3) of the continuity function is rewritten to the discrete-time representation by using sampling interval (observation interval) dt as shown in the formula (14), this treatment is the same for the least-squares method.

$$f(n) = Y0 + A1 \cdot \exp(-dt \cdot n/T1) + A2 \cdot \exp(-dt \cdot n/T2) + A3 \cdot \exp(-dt \cdot n/T3) + A4 \cdot \exp(-dt \cdot n/T4) + A5 \cdot \exp(-dt \cdot n/T5) \tag{14}$$

The state vector x_n is set as follow for convenience to calculate.

$$x_n^T = (x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11) = (A1, A2, A3, A4, A5, \exp(-dt/T1), \exp(-dt/T2), \exp(-dt/T3), \exp(-dt/T4), \exp(-dt/T5), Y0) \tag{15}$$

With these treatments, one-step forward estimation of x_n can be simplified a lot as there is no input for this calculation, and computational effort can be significantly reduced.

$$\hat{x}_n^- = f(\hat{x}_{n-1}^+) = \hat{x}_{n-1}^+ \tag{16}$$

Here, observation value y_n can be expressed by the formula (17).

$$\hat{y}_n^- = h(\hat{x}_n^-) = (\hat{x}_1^- \times \hat{x}_6^n + \hat{x}_2^- \times \hat{x}_7^n + \hat{x}_3^- \times \hat{x}_8^n + \hat{x}_4^- \times \hat{x}_9^n + \hat{x}_5^- \times \hat{x}_{10}^n + \hat{x}_{11}^-) \tag{17}$$

Based on the above established formulae, we tried to verify the adequacy of the expanded Kalman-filter by performing the exponential curve fitting with the expanded Kalman-filter method. The result was compared with the calculation by the Levenberg-Marquardt method under the same condition.

6. COMPARISON OF THE CALCULATION RESULT OF THE LEAST-SQUARES METHOD AND THE KALMAN-FILTER

In making comparative verification of the calculation results of the least-squares method and with those of the Kalman-filter, next relaxation data of over-voltage $dt=20$ s and $N=901$ (total 18,000 s=5 hrs) are as shown in figure 4.

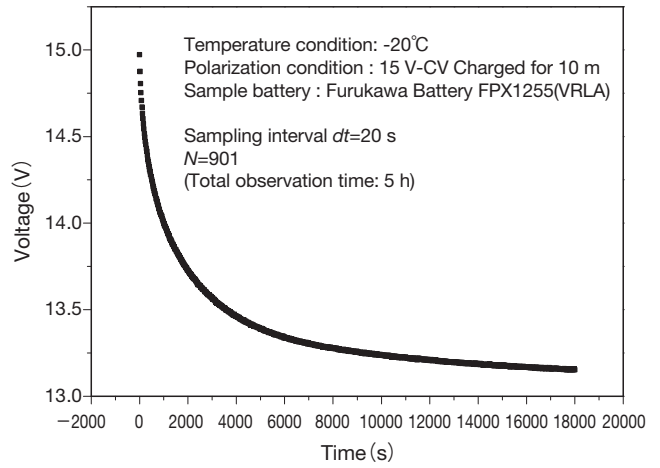


Figure 4 Relaxation data for trial.

The least-squares method that serves as a reference for comparison used data analysis software on the market OriginPro8.5 implementing the Levenberg-Marquardt method.

The learning result by OriginPro8.5 is shown in Table 1.

Table 1 Result of least square fitting.

	A1	A2	A3	A4	A5	T1	T2	T3	T4	T5	Y0
Optimization coefficient	0.18237	0.4074	0.58806	0.34749	0.36972	55.83597	352.1912	1985.511	1985.694	11750.23	13.07268

In performing the expanded Kalman-filter calculation, our self-made tool uses Excel and VBA macro programs. Both the least-squares method and the Kalman-filter have initial value dependence in calculation results. In this verification, following initial values are set, A1=0.3, A2=0.3, A3=0.3, A4=0.3, A5=0.3, T1=10, T2=100, T3=100, T4=1000, T5=10000, Y0=135. At first, as a straightforward way, the calculation result of one series of observation, started from $n=1$ to $n=901$ is shown in Table 2.

Table 2 Result of Kalman-filter without iteration.

	A1	A2	A3	A4	A5	T1	T2	T3	T4	T5	Y0
Optimization coefficient	0.243151	0.537924	0.477655	0.551855	0.125053	63.86299	423.646	4190.568	5308.926	6181.159	13.12579

Comparing Table1 and Table 2, it is difficult to say that similar results were obtained. To see the optimization progress in the process of observation, changes in Y0, which is the coefficient to represent OCV, is shown in Figure 5.

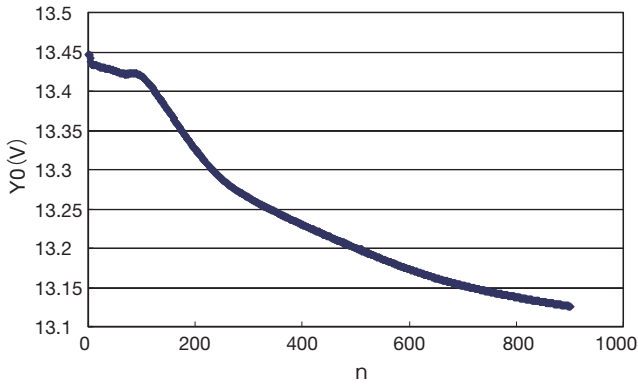


Figure 5 Optimization behavior of Y0 parameter.

From Figure 5, unfortunately in 5 hours of observation from $n=1$ to 901, T_0 is still in the process of value changing and not converged yet. As a matter of fact, with longer time of learning the calculation result goes towards the convergence, 5 hours of learning is almost the limit of the stable secured time for vehicles. From the original objective point of view, the stable OCV prediction in a short while, a longer time of observation than this time is not realistic any more. As a result of survey to find out the problem solving method, we have reached to the study used in the construction field. The optimization study for the full elastic-plastic spring approximation model of the ground in rapid loading test of piles, after N-th of the last filtering completion, the return to the initial observation value by using the obtained optimum values as the initial value, and repeat the calculation like the least-squares method⁵⁾. This method kills the original feature of the infinite response filter which does not require buffering of the observed value. Also computational effort is increased. Even computational effort is significantly increased by the repetition of k ; we decided to carry out the similar calculation considering that the most important thing is to see if the aimed curve fitting can be successfully performed by using the Kalman-filter. In the former study, the method to increase the covariance matrix by 20 times was taken at the repetition to return to the initial observation point. We repeated this treatment. The study result is shown in Figure 6 as the change followed to the repetitive calculation of Y_0 .

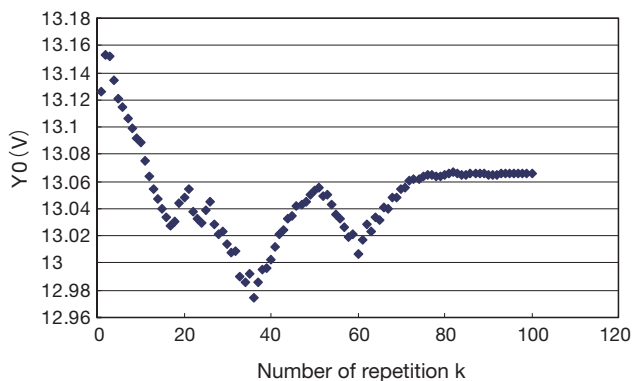


Figure 6 Optimization of Y0 parameter through iteration.

As shown in Figure 6, Y_0 converged by repeating calculation for 70 to 80 times. But, the feature of the Kalman-filter, infinite response filter, was broken. Furthermore, the effect of computational effort reduction would almost vanish because of the 70 to 80 times of repetitive calculations. Further consideration was made looking for an improvement method and the influence by a linear simplification in the sampling of the discrete-time interval of the expanded Kalman-filter. The image of the linear simplification in the expanded Kalman-filter is shown in Figure 7.

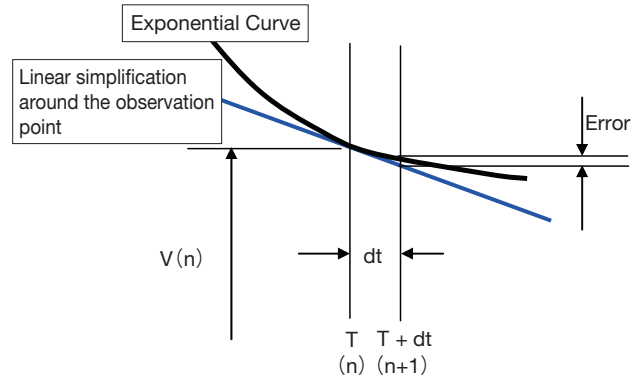


Figure 7 Image of linear simplification for non-linear system.

Even in a non-linear system, to explain behaviors in the short enough time dt , the linear simplification (Taylor series simplification of the primary) is possible as shown in Figure 7; this is widely used in general engineering field. The expanded Kalman-filter uses this method as well. In other words, the expanded Kalman-filter is the calculation method which can be satisfied only in the short enough sampling interval that satisfies the above simplification. Then, the shorter the sampling interval the less the linear simplification error, but the computational effort is increased as a matter of fact because of the increment in the observation points. And this would defy the purpose. The degree of error generated by the sampling interval for primary exponential function $X(n) = \exp(-dt \cdot n/T_0)$ is shown in Table 3, as an example.

Table 3 Error caused by linear simplification.

	dt			
	T_0	$T_0/2$	$T_0/5$	$T_0/10$
Error	$0.37 \times X(n)$	$0.107 \times X(n)$	$0.019 \times X(n)$	$0.005 \times X(n)$

As shown in Table 3, the linear simplification error of the exponential function can be calculated uniquely as a ratio to the power. The sampling interval dt of this study is 20 s. On the other hand, the smallest power coefficient T_1 of the optimization coefficient is 55.83597, this makes $dt = T_1/2.791799$ by the expression according to Table 3,

and we can see that close to 10% of error is generated. The feature of the Kalman-filter is established based on the assumption that the distribution of the error follows a Gaussian distribution in consideration of the influence of the error. But, in this case, the absolute value is large and in addition the above written error does not follow the Gaussian distribution. Meanwhile, in this case, the error caused from the power coefficient can be calculated as shown above Table 3. Here, we made the following trial for convergence improvement. The predictive calculation of the error was calculated from the power coefficient in the learning process. And the influenced part was subtracted by the observed noise.

In addition to the above matter, based on the 5 hours of data that was used this time, it was suggested that the 5th-order exponential decay function is in over-fitting because of the excessive number of terms, as T3 and T4 have the same number in the learning result of the least-squares method shown in Table 1. So, the change to the 4th-order exponential decay function was made together with the reduction of the number of terms. The recalculation result based on the above written two improvements is shown in Figure 8 and Table 4.

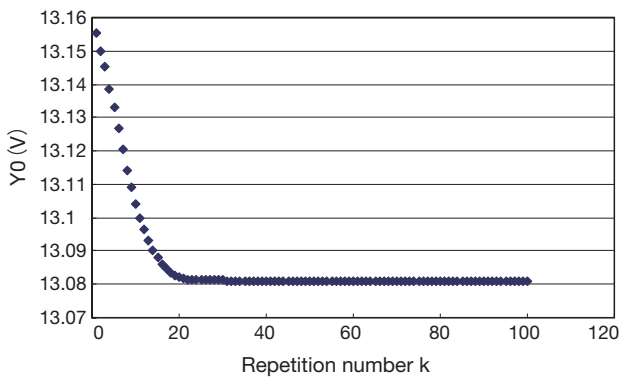


Figure 8 Optimization of Y0 in the modified calculation.

Table 4 Comparison of fitted coefficients of exponential function.

	A1	A2	A3	A4	T1	T2	T3	T4	Y0
Least-squares method	0.18237	0.4076	0.93555	0.36972	55.83597	352.1912	1985.579	11750.23	13.07268
Kalman-filter	0.304729	0.456292	0.950762	0.381907	31.15547	302.3189	1909.384	10775.69	13.08112

As shown in Figure 8, the learned value of Y0 is mostly converging by about 20 times of repetitive calculation; a significant repetition number reduction effect was obtained in comparison to Figure 6. With this degree of repetition number, a meaningful computational effort reduction is expected in comparison to the least-squares method. Furthermore, as shown in Table 4, the learned value of each coefficient was obtained as almost the same value of the calculated result by the least-squares

method. We consider that meaningfulness of the Kalman-filter calculation was confirmed as an optimized value as well.

7. CONCLUSION

In this report, we confirmed the effectiveness of the expanded Kalman-filter method as one mathematical method for purely curve fitting, and showed that the expanded Kalman-filter method can function as an effective tool by adding some ingenuity and improvement in the application. As a matter of fact, this study is not the final and we believe still there is room for improvement and optimization.

This is the same for the least-squares method, enough studies are necessary not only from a point of view of computational effort, but also from a stability in calculation of results.

Both the least-squares method and the Kalman-filter method have in common a dependence on the initial values of the solution, to be set at the beginning of the calculation, in the computational effort and stability. Of course, it is more advantageous when the initial values are closer to the optimum solution. In this study, the initial values were initially fixed, but initial value setting is a very important subject in the actual algorithm.

In addition, tools for curve fitting and coefficient optimization have been studied by many researchers on a daily bases, and many methods have been devised and proposed. We believe, it is very important to absorb and adopt these techniques widely for the establishment of an ideal algorithm.

REFERENCES

- 1) Noriyasu Iwane: "Battery Monitoring Sensor", Furukawa Denkou Jihou No.120 (in Japanese)
- 2) Kenichi Kanaya: "Optimum Mathematics Can be Understood Now on" KYORITSU SHUPPAN CO., LTD. (in Japanese)
- 3) Toru Katayama: "New Edition Applied Kalman-filter" Asakura Publishing Co., Ltd. (in Japanese)
- 4) Watanabe (Tadashi), Kanemura, Masuda, Watanabe (Masayoshi): "Electrochemistry, Basic Science Course" MARUZEN CO., LTD. (in Japanese)
- 5) Toshihiko Asou, Shinji Aramaki, Kiyoshi Uno, Tadayoshi Soda, Hidetoshi Ochiai: "A Study on the Application of the Kalman-filter to the Rapid Load Test of Piles" Journal of Japan Society of Civil Engineers No.673/III-54, 133-141. (in Japanese)