Contribution to the achievement of SDGs
by Furukawa Electric Group

Nelder-Mead Optimization for Optical Device Design: Raman Amplifiers and Hollow-Core Fibers

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ABSTRACT The investigations in this paper apply the Nelder-Mead simplex optimization algorithm, using MATLAB's **fminsearch** function, to improve the design of Raman amplifiers achieving a flat on-off gain profile and anti-resonant hollow-core fibers minimizing their confinement- and scattering loss values. Specialized solvers were developed for each application and integrated with the optimization algorithm. Results demonstrate that this approach effectively identifies optimal design parameters, leading to significant improvements in gain flatness for Raman amplifiers and substantial reductions in confinement- and scattering losses for hollow-core fibers. This study highlights the effectiveness of the Nelder-Mead method for optimizing complex optical device designs through simulation.

1. INTRODUCTION

Optimization is a critical component in solving numerous technical challenges, particularly those analyzed through computer simulation. In these simulations, the goal is often to identify the ideal configuration of a system, represented by a set of parameters, that yields to the best possible outcome. Achieving this requires the use of optimization algorithms. These algorithms iteratively adjust the parameter set, guided by a predefined strategy, to navigate the multi-dimensional parameter space and ultimately pinpoint the locations corresponding to optimal performance. This could mean finding either the minimum or maximum of an objective function depending on the specific problem.

A key advantage of this approach is the modularity it affords: the optimization algorithm can be implemented as a separate, independent module from the core simulation code. This separation is possible because the optimization algorithm interacts with the simulation solely through inputs and outputs. It acts as an external controller, adjusting the input parameters fed into the simulation. The simulation, in turn, produces an output that is evaluated by a specially designed 'merit function,' also known as an 'objective function.' This function quantifies the performance of the simulation for a given set of parameters, providing a single numerical value that represents the quality of the results. The optimization algorithm then uses this feedback from the merit function to iteratively refine the parameters, aiming to improve the performance in subsequent simulation runs.

To illustrate the practical application of this optimization approach, consider our Raman Amplifier simulation tool[1)], which leverages the principles discussed above. This tool models the Stimulated Raman Amplification (SRA) effect to calculate the amplification of multiple signal channels in an optical fiber. In this context, achieving a flat gain profile across all signal channels is crucial for optimal performance in telecommunication systems. However, manually adjusting the power and wavelength of the pump channels to achieve this flatness is a complex and challenging task. The interdependence of these parameters creates a multi-dimensional optimization problem, making a trial-and-error approach extremely inefficient and time-consuming. Without an automated optimization method, achieving an acceptable gain flatness would require an extensive, if not prohibitive, number of simulations runs and manual adjustments.

Another compelling example arises in the design of anti-resonant hollow-core optical fibers, specifically in the simulation of their loss characteristics. These fibers, which guide light within a hollow core surrounded by a specially designed cladding structure, exhibit both confinement loss and scattering loss but these losses altogether can be lower than the total loss of the lowest loss silica fiber[2), 3)]. Accurately modeling these fibers requires calculating the mode profile and its corresponding complex eigenvalue, the real part of which determines the propagation constant, and the imaginary part yields the confinement loss. A primary goal in fiber design is to minimize losses for the fundamental mode, specifically confinement and scattering losses, thereby improving transmission efficiency[4)]. Simultaneously, it is crucial to maxi-

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mize the loss of higher-order modes, or at least ensure their losses surpass a specific high threshold, to effectively suppress their propagation and maintain single-mode operation. To achieve this, optimization of the fiber's geometrical parameters, such as the diameter of the capillaries that confine the mode in the core, the number of the cladding elements, or the thickness of the capillary walls, becomes crucial. Adjusting these parameters directly impacts the mode profile and, consequently, the loss characteristics. Therefore, a systematic optimization approach is necessary to effectively navigate the design space and identify fiber geometries that exhibit the possible smallest loss.

This paper demonstrates the application of the Nelder-Mead simplex optimization method[5), 6)] to address the two aforementioned challenges in optical device simulation: optimizing the on-off gain in Raman amplifiers and minimizing the loss in anti-resonant hollow-core fibers. Leveraging MATLAB's built-in **fminsearch** function, which implements the Nelder-Mead algorithm, we developed specialized solvers for both the Raman amplifier and the Helmholtz eigenvalue analysis of optical fibers. These solvers are called iteratively by **fminsearch** to identify the optimal parameter sets that yield to the desired performance characteristics. The remainder of this paper is organized as follows: Section 2 elucidates the fundamental principles of the Nelder-Mead simplex method. Section 3 presents the optimization results for the Raman amplifier, focusing on achieving a flat on-off gain profile. Section 4 details the structural optimization of anti-resonant hollow-core fibers, aiming to minimize the loss of the fundamental mode and maximize it for the higher order modes. Finally, Section 5 concludes the paper by summarizing the key findings and discussing potential future research directions.

2. THEORY

The Nelder-Mead algorithm uses special n dimensional polytopes with n+1 vertices so called simplex[5)]. The elements of the simplex are determined by the Nelder-Mead coefficients[6)]: reflection(ρ), expansion(χ), contraction(γ) and shrinkage(σ). In the original algorithm[5)], the parameters should satisfy:

$$\rho > 0\,, \quad \chi > 1, \quad 0 < \gamma < 1, \quad 0 < \sigma < 1 \tag{1}$$

In the implemented version, the parameters set up as follows:

$$\rho = 1\,, \quad \chi = 2, \quad \gamma = \frac{1}{2}\,, \quad \sigma = \frac{1}{2} \tag{2}$$

The method runs through the iteration process until the stopping condition is met. The k[th] step of the iteration looks like as follows[6)]:

1. First step is to order and label the vertices of the simplex, Δ_k as $x_1^{(k)}, \ldots, x_{n+1}^{(k)}$, such that:

$$f_1^{(k)} \le f_2^{(k)} \le \cdots \le f_{n+1}^{(k)} \tag{3}$$

where $f_i^{(k)}$ represents $f\left(x_i^{(k)}\right)$. The k[th] iteration step will generate a new simplex Δ_{k+1} with a different set of n+1 vertices. As the goal is to minimize f, we refer to $x_1^{(k)}$ as the best vertex, and $x_{n+1}^{(k)}$ as the worst vertex. The result of each iteration step is either:

   a. New simplex, replacement vertex instead of $x_{n+1}^{(k)}$.
   b. Shrink performed, n new points with $x_1^{(k)}$ forms the new simplex.

2. Reflection point $x_r^{(k)}$ calculation:

$$x_r^{(k)} = x_{avg}^{(k)} + \rho\left(x_{avg}^{(k)} - x_{n+1}^{(k)}\right) \tag{4}$$

where:

$$x_{avg}^{(k)} = \frac{1}{n}\sum_{q=1}^{n} x_q^{(k)} \tag{5}$$

is the centroid of the best n points. Then the evaluation of $f_r = f\left(x_r\right)$ is the next step. If $f_1 < f_r < f_{n+1}$ accept the new point and terminate the iteration step as a new, more optimal simplex is created.

3. Expansion if $f_r < f_1$, expansion point calculation by:

$$x_e^{(k)} = x_{avg}^{(k)} + \chi\left(x_r^{(k)} - x_{avg}^{(k)}\right) \tag{6}$$

Then evaluate $f_e = f\left(x_e^{(k)}\right)$, accept $x_e^{(k)}$ if $f_e < f_r$ else accept $x_r^{(k)}$ and terminate the iteration step.

4. Contraction if $f_n \le f_r$

   Two cases:

   a. Outside: If $f_n < f_r < f_{n+1}$

$$x_c^{(k)} = x_{avg}^{(k)} + \gamma\left(x_r^{(k)} - x_{avg}^{(k)}\right) \tag{7}$$

Then evaluate $f_c = f\left(x_e^{(k)}\right)$. If $f_c \le f_r$ accept $x_c^{(k)}$ and terminate the iteration step. Otherwise skip to step 5 (shrink).

   b. Inside: If $f_r < f_{n+1}$, perform an inside contraction:

$$x_{ic}^{(k)} = x_{avg}^{(k)} - \gamma\left(x_{avg}^{(k)} - x_{n+1}^{(k)}\right) \tag{8}$$

Then evaluate $f_{ic} = f\left(x_{ic}^{(k)}\right)$. If $f_{ic} \le f_{n+1}$ accept $x_{ic}^{(k)}$ and terminate the iteration step. Otherwise skip to step 5 (shrink).

5. Perform shrinking: New simplex is constructed by the new vertices as:

$$v_q^{(k)} = x_1^{(k)} + \sigma\left(x_q^{(k)} - x_1^{(k)}\right) \tag{9}$$

$q=2,\ldots,n+1$. The next simplex will consist of $x_1^{(k)}, v_2^{(k)}, \ldots, v_{n+1}^{(k)}$ vertices.

The original **fminsearch** function meant to process unconstrained problems, later it had been modified according to Ref. 7) to fit constrained problems as well. The function called **fminsearchbnd** modifies the input variables by sine transformation[7)] to fit in the provided upper and lower limit boundaries. We implement this algorithm in product design and development tasks with results to be presented in the forthcoming sections.

3. RAMAN AMPLFIER SOFTWARE

Information density of long-haul telecommunication system has been continuously increasing ever since new technological developments have been introduced such as 5G and 6G. The directives set higher quality expectations for every optical element in the transmission line including the widely used Raman amplifiers. The phenomenon behind Raman amplifiers is Raman scattering[8] which is a highly nonlinear process thus predicting that the propagation in such system is challenging. Simulation can be an extremely helpful tool[1] to design or further develop the existing amplifiers which meet the new expectations (<1 dB gain deviation). Please note that in this section we will focus on the utilization of the Nelder-Mead algorithm in our Raman Amplifier software, further theoretical background and the exact solved equations had been already reported[1].

Raman Amplifier software uses the Nelder-Mead method to obtain backward propagating pump starting value at the beginning of the fiber. This is a mandatory step during calculations as the equation system is integrated in forward direction. The simplified backward propagation method consist of two steps as follows:

1. Forward propagation (backward pumps also in forward direction).
2. Use the obtained power values at the end of the fiber for backward pumps as an initial guess value for iteration with the Nelder-Mead algorithm.

Generally, the sought value for each backward pump will be between its maximum power at the fiber end and zero, this will be the boundary constraint for the optimization. The merit function in this case is relatively simple, as it is shown in Figure 1 which is the pump power evolution along the fiber from the beginning of the fiber (left side) to the end of the fiber (right side). P_a^k represents the simulated value for the k[th] pump channel while P_b^k is the predetermined boundary value. The goal is to minimize ΔP by changing the starting value P_s^k for each pump. In our experience, the algorithm can minimize the difference far beyond the realistic point in the cost of computation time.

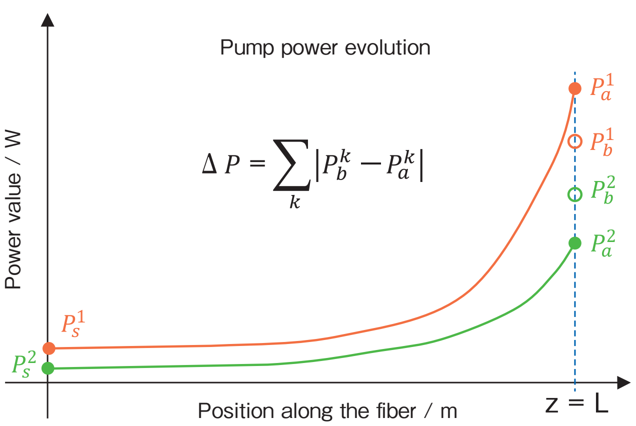

Figure 1 Backward pump power propagation during the Nelder-Mead iterations.

To maintain efficiency, an acceptable error was set up in the order of 0.1 mW, which means that every pump power can be adjusted to this precision.

To support the new generation Raman amplifier development, a pump power optimization was introduced in our Raman Amplifier software[1]. The goal is to provide a flat gain profile over a wide frequency range (multiple optical bands). To achieve this, a merit function has been determined which can be seen in Figure 2. The green curve represents the gain of the amplifier as function of the wavelength, T represents the target gain which can be either net gain or on-off gain. A represents the deviation range around T which is still accepted as a good result.

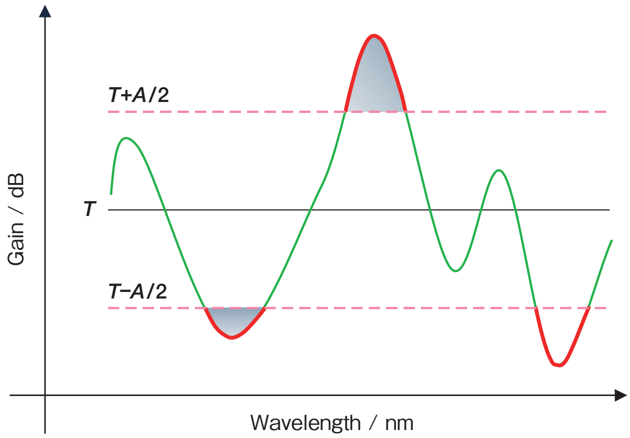

Figure 2 Raman amplifier gain profile around 'T' target gain.

The error to be minimized by the Nelder-Mead method is calculated as:

$$\mathrm{Error} = \sum_k e_k \tag{10}$$

where

$$e_k = |T - G_k| \tag{11}$$

if

$$G_k \notin \left[T - \frac{A}{2}, T + \frac{A}{2}\right] \tag{12}$$

else

$$e_k = 0 \tag{13}$$

where G_k represents the gain value for the k[th] signal channel. Utilizing this method, we selected six equally spaced steps from the steps of the entire optimization procedure that are presented in Figure 3. The target on-off gain is 8.1 dB while the accepted error is 0.2 dB. In the right-hand side figure, the results can be seen, presenting a gain profile with 0.191 dB maximal difference over 80 nm bandwidth. The realization of the simulation results can contribute to the foundation of next generation Raman amplifiers manufactured by Furukawa Electric Co., Ltd. (FEC) with exceptional gain deviation properties which is a requirement from modern amplifiers in the telecommunication bands.

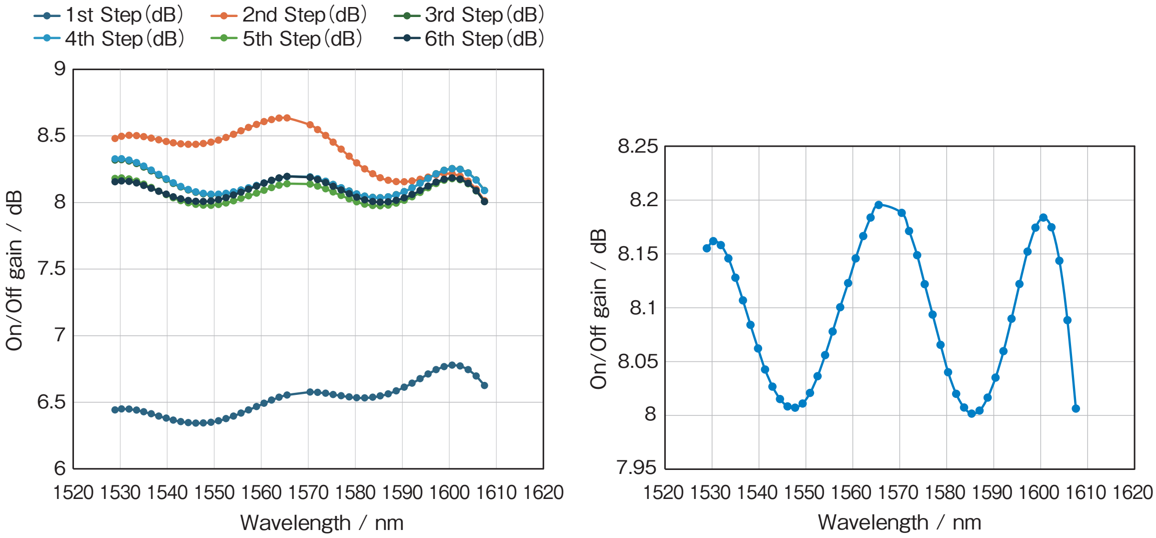

Figure 3 Optimization process in couple of steps (left), final on-off gain result (right).

4. OPTIMIZATION OF HOLLOW-CORE FIBERS

Recently extensive research has been done on antiresonant hollow-core fibers as their optical properties are starting to exceed regular solid-core fibers in every aspect. However, since material loss is negligible in these types of fibers, there are other new types of losses that govern the total loss of these kinds of fibers, namely confinement loss and scattering loss[4), 9)]. To keep the losses as low as possible, careful optimization of the fiber structure is necessary. Right now, the state-of-the-art antiresonant hollow-core fiber is a double-nested structure[2)] as shown in Figure 4.

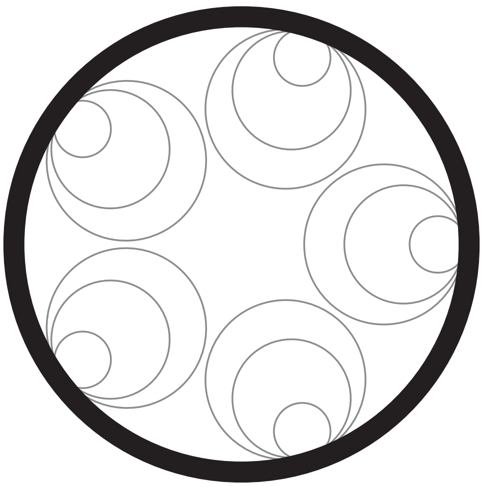

Figure 4 State of the art hollow-core fiber structure.

As it is double nested, they have three capillary rings in the structure. Each one has a size and width and if we also consider the size of the core that adds up to 7 parameters in total. With that number of variables and not significantly small range for each of the parameters, regular parameter sweep for the whole parameter space is quite challenging. However, utilizing a simplex optimization algorithm with the total loss of the fiber as a merit function can quickly and efficiently find a good local minimum in the loss curve. To ensure good efficiency and avoid unnecessary unrealistic cases, a modified version of the Nelder-Mead simplex algorithm, namely the **fminsearchbnd**[7)] Matlab function can be used here as well in that each parameter is limited to a specific range where the producibility of the fiber can be realized. An example for fiber optimization is shown in Figure 5.

The optimization started with random capillary parameters and a fixed core size. The results show that the simplex algorithm could reduce the loss really close to the state-of-the-art fiber in just a couple of hours. While this is a good result in the case of total loss, there are some other factors that determine the overall performance of an optical fiber, for example higher order mode loss. In this case, there are two higher order modes that are close to the fundamental mode in loss which degrades the performance of the fiber significantly. This problem can also be treated with the simplex optimization method, by choosing a proper merit function. In this following analysis, a negative effect for the higher order modes has been added to the returning error of the optimization.

$$\mathrm{Error} = \alpha_{LP_{01}} + C - \min\left(\alpha_{LP_{rs}}\right) \tag{14}$$

If

$$\min\left(\alpha_{LP_{rs}}\right) < C$$

else

$$\mathrm{Error} = \alpha_{LP_{01}}$$

where C is a constant loss value above which the higher order mode loss is expected and α is the loss of the LP_{rs} type modes where r and s are the azimuthal and radial mode number, respectively. With this form of the merit function, it effectively produces a fiber geometry with a low loss value for the fundamental (LP_{01}) mode and high loss values for the higher order modes (LP_{rs}).

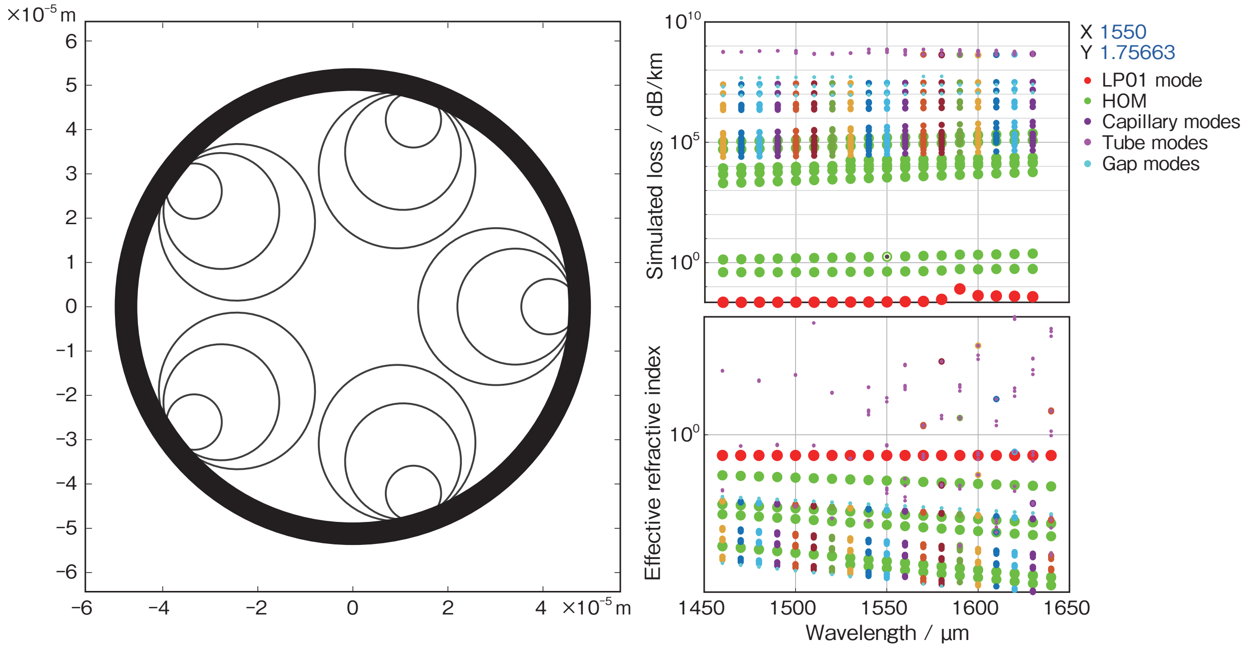

Figure 5 Results from simplex fiber optimization.

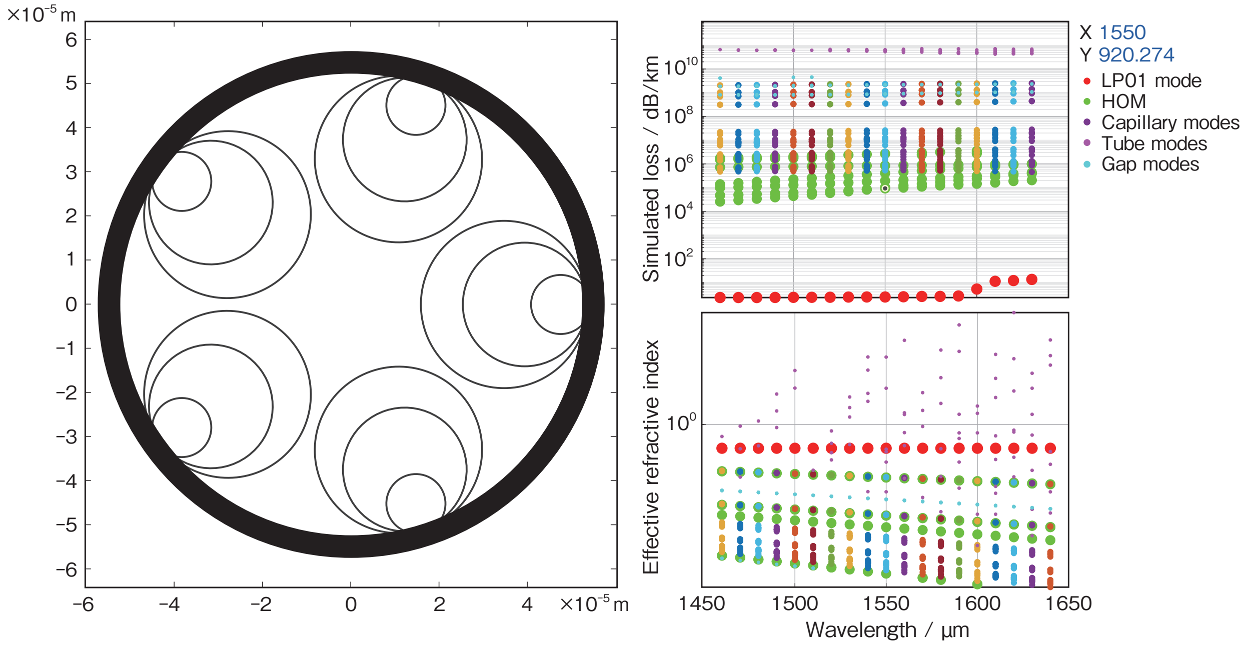

Figure 6 Analysis with modified merit function.

The obtained results in Figure 6 show much larger distance in loss between the fundamental and higher order modes while the fundamental mode loss is practically unchanged. To demonstrate the significant change in higher order mode loss, we depicted a data point in either Figure 5 and Figure 6 at 1550 nm and the ‘Y’ values are also presented in dB/km units.

This effect on the higher order modes is simply achieved by increasing the diameter of the two inner capillaries and decreasing the ratio between their diameters. However, one must be careful with the construction of the merit function, because if the loss of the higher order modes has too much influence on the returning error, the loss of the fundamental mode can increase significantly while having unnecessarily high higher order mode losses.

5. SUMMARY

We presented the utilization of the Nelder-Mead simplex optimization process in two key research areas in next generation telecommunications. Raman amplifier simulation results exceed the requirements providing FEC an edge in a highly competitive business area. The design process of a low loss hollow core fiber can be sped up by using a parameter optimization which furthermore provides knowledge and deeper understanding of the different structural arrangement in the fiber.

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