Four-Wave Mixing in Optical Fibers and Its Applications

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ABSTRACT Four-wave mixing (FWM) is a phenomenon that must be avoided in DWDM transmission, but depending on the application it is the basis of important second-generation optical devices and optical device measurement technology. This paper discusses the theory of FWM, and then introduces one of its applications --a broadband all-optical simultaneous wavelength converter developed using a high nonlinearity dispersion fiber (HNL-DSF) that efficiently produces FWM. The conversion bandwidth extends to 23.3 nm HWHM (half width at half maximum), the widest yet reported for wavelength conversion using non-polarization-maintaining fiber. As a further application, a novel technique is introduced for measuring the nonlinear coefficient of optical fibers by evaluating FWM generating efficiency. With this technique it is now possible to effect simultaneous measurement of the chromatic dispersion and nonlinear coefficient of fiber.

1. INTRODUCTION

When a high-power optical signal is launched into a fiber, the linearity of the optical response is lost. One such nonlinear effect, which is due to the third-order electric susceptibility is called the optical Kerr effect.[1), 2)] Four-wave mixing (FWM) is a type of optical Kerr effect, and occurs when light of two or more different wavelengths is launched into a fiber. Generally speaking FWM occurs when light of three different wavelengths is lauched into a fiber, giving rise to a new wave (know as an idler), the wavelength of which does not coincide with any of the others. FWM is a kind of optical parametric oscillation.

In the transmission of dense wavelength-division multiplexed (DWDM) signals, FWM is to be avoided, but for certain applications, it provides an effective technological basis for fiber-optic devices. FWM also provides the basic technology for measuring the nonlinearity and chromatic dispersion of optical fibers. This paper discusses those aspects of R & D into FWM applications that the authors have carried out recently in connection with broadband all-optical simultaneous wavelength conversion and a technique for measuring the nonlinear coefficient of optical fibers.

2. THEORY OF FWM

Figure 1 is a schematic diagram that shows four-wave mixing in the frequency domain. As can be seen, the light that was there from before launching, sandwiching the two pumping waves in the frequency domain, is called the probe light (or signal light). The idler frequency f_{idler} may then be determined by

$$f_{\mathrm{idler}} = f_{\mathrm{p1}} + f_{\mathrm{p2}} - f_{\mathrm{probe}} \qquad (1)$$

where: f_{p1} and f_{p2} are the pumping light frequencies, and f_{probe} is the frequency of the probe light.[1), 2)]

This condition is called the frequency phase-matching condition. When the frequencies of the two pumping waves are identical, the more specific term "degenerated four-wave mixing" (DFWM) is used, and the equation for this case may be written

$$f_{\mathrm{idler}} = 2f_{\mathrm{p}} - f_{\mathrm{probe}} \qquad (2)$$

where: f_{p} is the frequency of the degenerated pumping wave.

Continuous-wave DFWM may be expressed by the following nonlinear coupled-mode equations[1)]

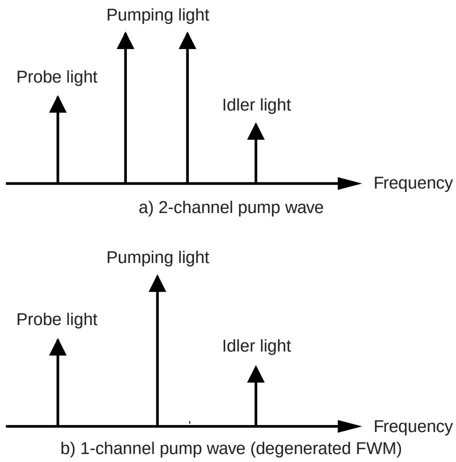

Figure 1 Schematic of four-wave mixing in the frequency domain

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$$\frac{dE_{\mathrm{p}}}{dz}+\frac{1}{2}\alpha E_{\mathrm{p}}=\mathrm{i}\gamma\left(\left|E_{\mathrm{p}}\right|^{2}+2\left|E_{\mathrm{probe}}\right|^{2}+2\left|E_{\mathrm{idler}}\right|^{2}\right)E_{\mathrm{p}}+2\mathrm{i}\gamma E^{*}{}_{\mathrm{p}}E_{\mathrm{probe}}E_{\mathrm{idler}}\exp(\mathrm{i}\Delta\beta z)$$

$$\frac{dE_{\mathrm{probe}}}{dz}+\frac{1}{2}\alpha E_{\mathrm{probe}}=\mathrm{i}\gamma\left(\left|E_{\mathrm{probe}}\right|^{2}+2\left|E_{\mathrm{idler}}\right|^{2}+2\left|E_{\mathrm{p}}\right|^{2}\right)E_{\mathrm{probe}}+2\mathrm{i}\gamma E^{*}{}_{\mathrm{idler}}E_{\mathrm{p}}{}^{2}\exp(-\mathrm{i}\Delta\beta z) \quad (3)$$

$$\frac{dE_{\mathrm{idler}}}{dz}+\frac{1}{2}\alpha E_{\mathrm{idler}}=\mathrm{i}\gamma\left(\left|E_{\mathrm{idler}}\right|^{2}+2\left|E_{\mathrm{p}}\right|^{2}+2\left|E_{\mathrm{probe}}\right|^{2}\right)E_{\mathrm{idler}}+2\mathrm{i}\gamma E^{*}{}_{\mathrm{probe}}E_{\mathrm{p}}{}^{2}\exp(-\mathrm{i}\Delta\beta z)$$

where: z is the longitudinal coordinate of the fiber, α is the attenuation coefficient of the fiber, and E_{p}, E_{probe} and E_{idler} are the electric field of the pumping, probe and idler waves.

γ is the nonlinear coefficient, and is obtained by[1)]

$$\gamma \equiv \frac{2\pi f_{\mathrm{p}}}{c}\cdot\frac{n_2}{A_{\mathrm{eff}}} \quad (4)$$

where: n_2 is the nonlinear refractive index, A_{eff} is the effective area of the fiber and c is the speed of light in a vacuum.

The term $\Delta\beta$ in Equation (3) represents the phase mismatch of the propagation constant, and may be defined as

$$\Delta\beta=\beta_{\mathrm{probe}}+\beta_{\mathrm{idler}}-2\beta_{\mathrm{pump}}=-\frac{8\pi f_{\mathrm{p}}{}^{2}}{c}D(f_{\mathrm{p}})(f_{\mathrm{probe}}-f_{\mathrm{p}}) \quad (5)$$

where: D is the chromatic dispersion coefficient.

To generate FWM efficiently, it is required that pump wavelength conincides with the fiber zero-dispersion wavelength.[3)] The first term on the right side of Equation (3) represents the effects of self-phase modulation (SPM) and cross-phase modulation (XPM) resulting from the optical Kerr effect.

3. WAVELENGTH CONVERSION BY FIBER FOUR-WAVE MIXING

3.1 Significance of Wavelength Converters

Wavelength converter is simply a device for converting the injected signal light from one wavelength to another.[8)–13)] It therefore is seen to have great promise in configuring the photonic networks of the future using optical cross connects. A number of methods of wavelength conversion have been proposed, of which parametric conversion using optical fiber FWM offers two major advantages: high conversion speed and the ability to effect simultaneous conversion of signals within a wavelength bandwidth.

3.2 Wavelength Conversion in the Fiber

The most important characteristics desired of wavelength converters using parametric conversion are high conversion efficiency and broad bandwidth.

To achieve this kind of wavelength conversion, the following conditions must be met:

(a) pump wavelength must coincide with zero-dispersion wavelength;
(b) chromatic dispersion variation in the longitudinal direction of the fiber should be minimized; and
(c) states of polarization of the pump and signals must coincide.

As has already been argued in the literature,[6), 7)] in order to broaden the conversion bandwidth, consideration must additionally be given to coherence length. The arguments concerning efficient DFWM generation may be summarized as follows: Letting Δf be the frequency spacing between the pumping light and the signal (or idler) light, fiber length L must, to produce effective DFWM across the frequency band, satisfy the condition

$$L \le L_{\mathrm{coh}} \equiv \frac{2\pi}{\left|\Delta\beta\right|}=\frac{c}{4\pi^{2}f_{\mathrm{p}}\left|D(f_{\mathrm{p}})\right|}\cdot\frac{1}{\Delta f^{2}} \propto \frac{1}{\Delta f^{2}} \quad (6)$$

where: L_{coh} is coherence length, a parameter having a length dimension.

As Equation (6) shows, fiber length must be reduced to effect broadband simultaneous wavelength conversion at large values of Δf. Reducing fiber length is also significant in terms of condition (b), since it results in a homogeneous chromatic dispersion distribution along the fiber. Reducing fiber length is also effective in satisfying condition (c). Unless polarization-maintaining fiber (PMF) is used, the state of polarization at launching is not maintained until output. This is due to variations in polarization in the length direction caused by birefringence within the fiber. Even if the state of polarization is aligned at the time of launching into the fiber, the relative phase difference between the pumping light and the signal light can be expressed, if birefringence Δn is present, as

$$\Delta\phi=\frac{2\pi}{c}\Delta n\cdot\Delta f\cdot L \quad (7)$$

One way of achieving a broader conversion band Δf is to reduce Δn. It has been reported[12)] that broadband simultaneous wavelength conversion, with a Δn of effectively zero at 36.0 nm HWHM has been successfully achieved taking advantage of DFWM in the eigenstate of polarization using PMF. If, however, fiber length L is reduced, even the limited Δn can to some extent control the problem of mismatching of polarization.

If the fiber is shortened, however, its length will be insufficient to produce nonlinear interactions. To compensate for this, it was decided to use HNL-DSF.

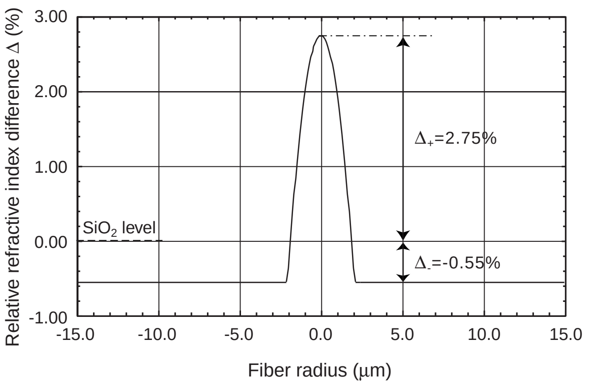

Figure 2 Refractive index profile of HNL-DSF

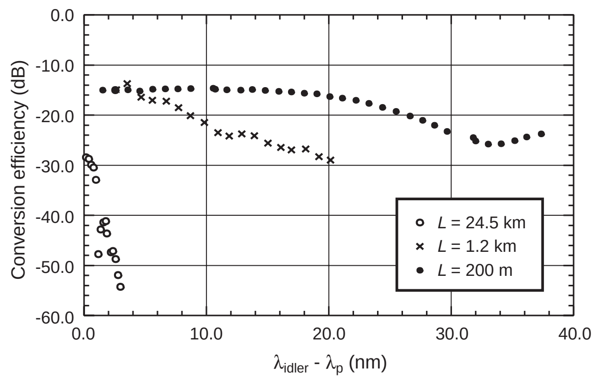

Figure 4 Measured values of conversion efficiency for fibers of selected lengths

Table 1 Transmission Characteristics of HNL-DSF

Characteristic	Measured value
Attenuation coefficient	0.61 dB/km
Zero-dispersion wavelength	1565.5 nm
Dispersion slope (at zero-dispersion wavelength)	0.029 ps/nm^2/km
Nonlinear coefficient	13.8 W^{-1}km^{-1}

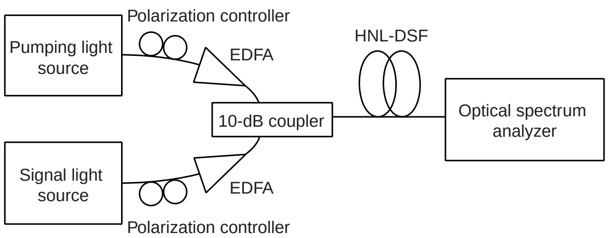

Figure 3 Setup for wavelength conversion experiment

4. EXPERIMENTS IN BROADBAND SIMULTANEOUS ALL-OPTICAL WAVELENGTH CONVERSION USING HNL-DSF

Figure 2 shows the refractive index profile of the HNL-DSF used in these experiments, and Table 1 shows transmission characteristics. The fiber was made by vapor-phase axial deposition, and had a nonlinear coefficient γ of 13.8 W^{-1}km^{-1}, approximately five times the value for ordinary DSF. Figure 3 shows the experimental setup.

Both the pumping and probe (signal) were continuous waves. The lightwaves amplified by the erbium-doped fiber amplifiers (EDFAs) were coupled using a 10-dB coupler. There are polarizers at the output terminal of the coupler, and the states of polarization of the pumping and signal at input into the HNL-DSF are in alignment. The output was measured by an optical spectrum analyzer to find idler optical power. In this way it was possible to find conversion efficiency G_c, which may be stated as

$$G_c = \frac{P_{\text{idler}}(z=L)}{P_{\text{probe}}(z=0)} \tag{8}$$

Figure 4 shows the measured values of conversion efficiency obtained for fibers 24.5, 1.2 and 0.2 km in length. During measurement, the pumping wavelength was made to agree with the zero-dispersion wavelength of the fiber. The injected pumping power was set at 100 mW (20 dBm), and signal power was 1 mW (0 dBm).

From Figure 4 it can be seen that as the length of the HNL-DSF is reduced, the bandwidth broadens, reaching 23.3 nm HWHM at a length of 200 m--the greatest bandwidth heretofore achieved using non-polarization maintaining fiber. [13]

5. MEASUREMENT OF NONLINEAR COEFFICIENT AND CHROMATIC DISPERSION

5.1 Nonlinear Coefficients

The explosive growth in long-haul telecommunications achieved in recent years has been largely attributable to DWDM technology and the role played by EDFAs, [14] but the nonlinear effects of signals amplified by EDFAs have resulted in the degradation of system performance.

Attention has recently been focused on dispersion managed systems as a means of suppressing FWM. [15] Reverse-dispersion fiber (RDF) is used in combination with conventional single-mode fiber (SMF). [16] At 1550 nm, RDF has a chromatic dispersion of the same magnitude as SMF but of opposite sign (normal dispersion), and the dispersion slope is reversed. Thus it can compensate for both dispersion and dispersion slope simultaneously. The results of high-capacity WDM experiments using dispersion-managed systems consisting of SMF and RDF have been reported. [17, 18]

A number of methods have been developed for measuring the nonlinear coefficient γ, including the use of self-phase modulation [19], cross-phase modulation [20] and four-wave mixing. [21, 22] In the present paper a technique was considered that was applicable to a comparatively wide normal dispersion domain, and yet measurements could be carried out by all-optical means. [23, 24] This was because it was realized that as dispersion-managed systems become more widely used and the demand for RDF and other fiber having normal dispersion increases, so will the need to evaluate it.

5.2 Principles of Measurement

Let us discuss measurement in terms of the pump undepleted approximation proposed by Stolen and Bjorkholm,[1), 7)] which in the case of DFWM is accomplished by Equation (9).

This is an approximation in which the attenuation coefficient α of Equation (3) is zero and the pumping power is taken to be so large as to be dominant. For this reason the pumping light is not subject to DFWM-induced reaction. The signal light and idler light are of about the same magnitude, and interact together through DFWM.

$$\frac{dE_{\rm p}}{dz} = {\rm i}\gamma \left|E_{\rm p}\right|^2 E_{\rm p}$$

$$\frac{dE_{\rm probe}}{dz} = 2{\rm i}\gamma \left|E_{\rm p}\right|^2 E_{\rm probe} + 2{\rm i}\gamma E^*_{\rm idler} E_{\rm p}{}^2 \exp(-{\rm i}\Delta\beta z) \quad (9)$$

$$\frac{dE_{\rm idler}}{dz} = 2{\rm i}\gamma \left|E_{\rm p}\right|^2 E_{\rm idler} + 2{\rm i}\gamma E^*_{\rm idler} E_{\rm p}{}^2 \exp(-{\rm i}\Delta\beta {\rm z})$$

Solving Equation (9) analytically, conversion efficiency $G_{\rm c}$ in the normal dispersion domain of the fiber may be represented[1)] as

$$G_{\rm c} = \gamma^2 P_{\rm p}{}^2 L^2 \left[\frac{\sin(gL)}{gL}\right]^2 \qquad (10)$$

wherein g is termed parametric gain, and can be obtained by

$$g \equiv \sqrt{\frac{1}{4}\Delta\beta(\Delta\beta + 4\gamma P_{\rm p})} \qquad (11)$$

The following is an explanation of the principles of measurement using the above terms.

If in Equations (10) and (11) $P_{\rm p}$ is a variable and fiber length L is known, γ and $\Delta\beta$ are the unknowns. It is possible to find by measurement the two conversion efficiencies $G_{\rm c}$ corresponding to two different values of pumping power $P_{\rm p}$. Mathematically, this may be regarded as obtaining two simultaneous equations with respect to the two unknowns γ and $\Delta\beta$. Solving this simultaneous equation yields the nonlinear coefficient and the chromatic dispersion. Actually, to minimize unavoidable measurement errors as much as possible, conversion efficiency $G_{\rm c}$ was found for successive values of pumping power without solving the equation, and γ and $\Delta\beta$ were obtained by the Levenberg-Marquardt least square method.[25)]

6. MEASUREMENT OF NONLINEAR COEFFICIENT OF RDF

6.1 System Setup

Figure 5 shows the experimental setup. It is substantially the same as that shown in Figure 3, except that a bandpass filter is used to reduce the amplified spontaneous emission of the EDFA amplifying the pump. Also a 15-dB coupler is used to couple the probe light and pumping light. Since the polarization controller (PC) is positioned after the EDFA, an attenuator is used. This results in a reduction in the power of the pumping light after coupling,

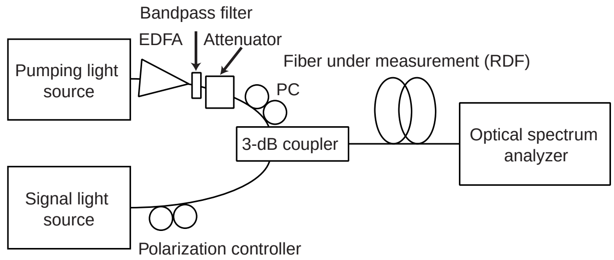

Figure 5 Setup for simultaneous measurement of nonlinear coefficient and chromatic dispersion

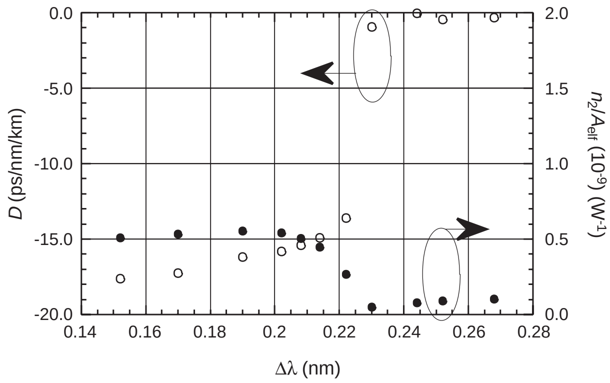

Figure 6 Measured values of nonlinear coefficient $n_2/A_{\rm eff}$ (•) and chromatic dispersion D (o) for 10-km fiber with a pumping wavelength of 1553.0 nm

so it is input into the fiber under measurement with the states of polarization of the probe light and pumping light carefully aligned and without the use of a polarizer (see Figure 3). Measurements of input probe light were taken with an optical power meter and of output power with an optical spectrum analyzer (OSA) having 0.01-nm resolution, to find the conversion efficiency.

6.2 Optimizing Measurement Conditions

To make an accurate evaluation of γ and $\Delta\beta$ using Equations (10) and (11), it was found necessary to give some consideration to the measurement conditions because: a) pumping power had to be operative below the stimulated Brillouin scattering (SBS) threshold value determined by the fiber under measurement and the line width of the pumping light source; and b) the optical power of the probe was set 25 dB lower than pumping power. This was to ensure the assumption that in a DFWM system using Equation (9), in which pumping light is assumed to be dominant.

Measurements and evaluations were then made at the two conditions described above. Measurements were carried out on an RDF having a total length of 10 km. The pump wavelength was set at 1553 nm. Specifically, $n_2/A_{\rm eff}$ was evaluated from γ using Equation (4) and chromatic dispersion coefficient D was evaluated from $\Delta\beta$ using Equation (5). Figure 6 shows the results obtained. The horizontal axis shows the wavelength spacing $\Delta\lambda$ between the pump and probe, and the vertical axis shows the corresponding measured value.

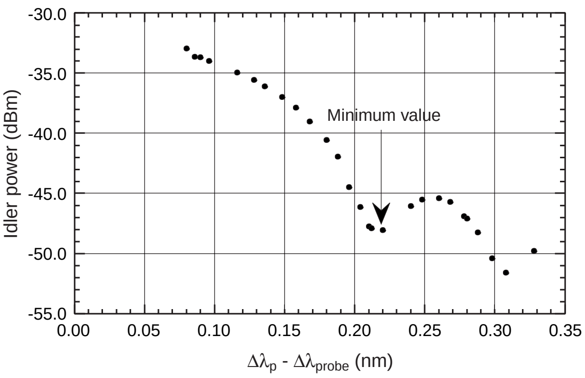

Figure 7 Relationship of idler power to probe wavelength for a 10-km fiber with a pumping wavelength of 1553.0 nm

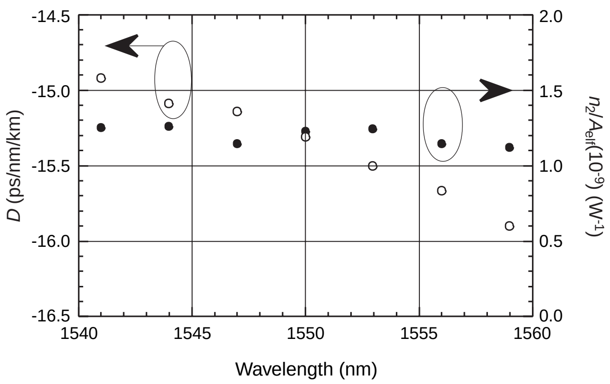

Figure 8 Measured values of nonlinear coefficient n_2/A_{eff} (•) and chromatic dispersion D (o) for 830-m RDF

As can be seen from Figure 6, the values of n_2/A_{eff} generally depend on λ, and the value changes greatly at $\Delta\lambda$ of about 0.22 nm. When the chromatic dispersion coefficient for identical fiber was measured independently by the phase-shift method, it was found to be 15.45 ps/nm/km at a wavelength of 1553 nm, demonstrating that at values of $\Delta\lambda$ greater than 0.22 nm, accurate evaluation was not obtained. This means that approximation by means of Equation (10) cannot be applied.

We measured the idler power by changing the probe wavelength while fixing the pump wavelength. The result is shown in Figure 7. It is known that the conversion efficiency decreases as $\Delta\lambda$ increases. The conversion efficiency has a minimal value when $\Delta\lambda$=0.22 nm, after which it increases as $\Delta\lambda$ decreases. Equation (6), which was introduced in the discussion of coherent length, can also be considered, in determining fiber length, as limiting the bandwidth, This limited bandwidth corresponds to the minimum value in Figure 7. [1] In other words this technique is useless unless $\Delta\lambda$ is less than 0.22 nm.

Based on Figure 6, the value of $\Delta\lambda$ evaluated as optimum was 0.21 nm. These results demonstrated that unless the optimal value was selected for wavelength spacing $\Delta\lambda$, the evaluation would include errors, so that measurements corresponding to those in Figure 7 were carried out for all fibers. This showed the need to take the measurements and carry out evaluations at the approximate value of $\Delta\lambda$ that yielded the first minimum value of conversion efficiency as $\Delta\lambda$ was increased, and this conclusion was confirmed by measurements made using a number of different RDFs.

6.3 Results of RDF Measurements

Measurements were made using RDFs of four different lengths: 0.83, 5, 10 and 20 km, at pumping wavelength increments of 3 nm. Figure 8 shows the results, from which the dispersion slope was obtained.

Table 2 compares the results of evaluations of n_2/A_{eff} for the four RDFs measured with results obtained independently by cross-phase modulation (XPM). Similarly the results of evaluations of chromatic dispersion and dispersion slope are shown against those made by the phase-shift method.

Table 2 Comparison of simultaneously measured values of nonlinear coefficient n_2/A_{eff} with those by cross-phase modulation, and of chromatic dispersion D with those by the phase-shift method in RDFs

	Nonlinear coefficient n_2/A_{eff} (W^{-1})		Chromatic dispersion @ 1550 nm (ps/nm/km)		Dispersion slope @ 1550 nm (ps/nm²/km)	
Fiber length L (km)	by 4WM	by XPM	by 4WM	by PSM	by 4WM	by PSM
0.83 km	1.200	1.20	-15.36	-15.30	-0.053	-0.049
5 km	0.764	1.19	-15.05	-15.30	-0.042	-0.049
10 km	0.520	1.17	-15.35	-15.30	-0.048	-0.049
20 km	0.366	1.39	-15.11	-15.10	-0.039	-0.024

The values for chromatic dispersion and dispersion slope for different wavelengths were in substantial agreement with the values measured by the phase-shift method. For n_2/A_{eff}, on the other hand, it was found that error gradually increased with fiber length. This is attributed to the failure to account for the effects of the attenuation coefficient α, which cannot be ignored at longer fiber lengths, in the approximation using Equation (9).

6.4 Discussion Relating to Long-Length Fibers

In applying the method described above to long-length fibers, the attenuation coefficient has to be taken into account. For this reason an approximation, in which pumping light and probe light are attenuated independently of homogeneously with DFWM has been developed [26] and may be expressed by

$$G_c = \gamma^2 P_p^2 L^2 \exp(-3\alpha L)\left[\frac{\sin(gL)}{gL}\right]^2 \quad (12)$$

and

$$g \equiv \sqrt{\frac{1}{4}\Delta\beta(\Delta\beta + 4\gamma P_p e^{-\alpha L})} \quad (13)$$

When the evaluation was repeated using these equations, it was confirmed that the value of n_2/A_{eff} agreed with the value obtained by XPM, irrespective of fiber length.

7. CONCLUSION AND OUTLOOK FOR THE FUTURE

In this paper the authors have examined techniques for achieving broadband all-optical simultaneous wavelength conversion by taking advantage of four-wave mixing (FWM) occurring in the fiber, together with techniques for the simultaneous measurement of the nonlinear coefficient and chromatic dispersion.

It has been demonstrated that the use of short-length HNL-DSF simultaneously solves the problems of chromatic dispersion variance along the longitudinal direction and polarization mismatch of probe and pump. It has been experimentally demonstrated that simultaneous wavelength conversion is possible over a bandwidth of 23.3 nm, the widest for non-polarization-maintaining fibers.

The authors have developed a technique for measuring the nonlinear coefficient without electrical signal processing by combining DFWM technology with the least square method for nonlinear functions. Measurement conditions have been optimized for the application of this technique, and it has been demonstrated that simultaneous measurement of nonlinear coefficient and chromatic dispersion are possible under these optimized conditions. The values obtained are in good agreement with those obtained using the conventional XPM and phase-shift methods. The present technique should also, in theory, be applicable to the anomalous dispersion domain and in the vicinity of zero-dispersion.

ACKNOWLEDGMENTS

The authors wish to thank Y. Suzuki, T. Yagi, R. Sugizaki, S. Arai and K. Mukasa of Fiber Development Center for providing HNL-DSF and RDF. We also thank H. Ogoshi for fruitful discussions. Last but not least, we thank H. Miyazawa for his continuous encouragement.

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Manuscript received on October 18, 1999.